UNIT 4

POWER TRANSMISSION

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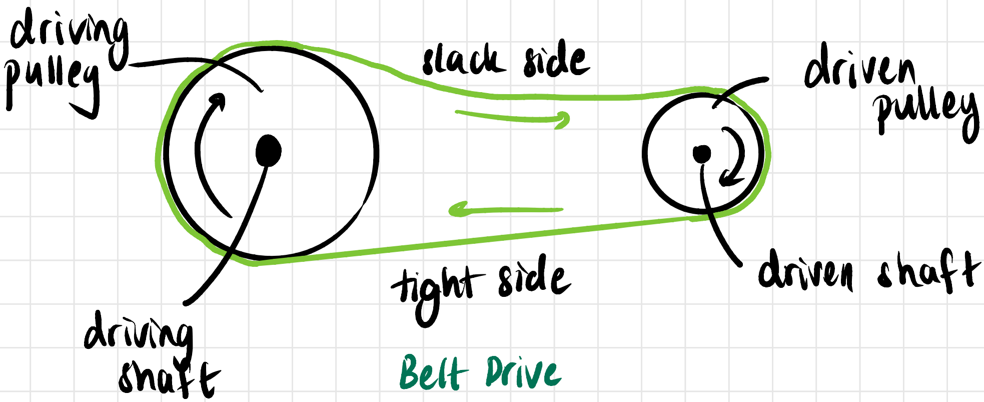
Transmission systems

1. Belt Drives
2. Rope Drives
3. Chain Drives
4. Gear Drives

Belt Drives

- 2 pulleys
- Belt encircles them
- rotary motion transmitted
- driving pulley ⟶ driven pulley
- force: friction



Belt Drive

- Direction of rotation determines slack/tight sides
- Tension depends on angle of contact
- Slip may cause driven pulley to rotate slowly

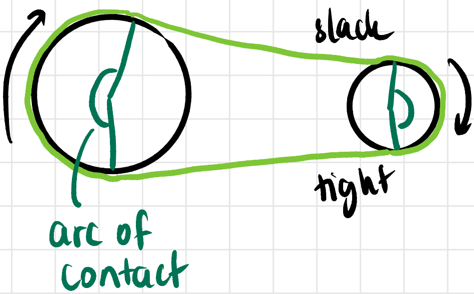
Materials

1. Leather — wet & dry
2. Rubber — typically dry
3. Canvas — when atm interferes with leather/rubber
4. Balata — cotton + balata

Types of Belt Drives

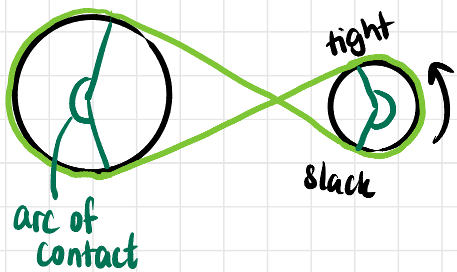
1. Open
2. Crossed

Open Belt Drive



- same direction
- never vertical
- diff. angles for diff sizes
- less belt length

Crossed Belt Drive



- at cross, wear & tear
- opposite directions
- same angle
- can be vertical
- more length required

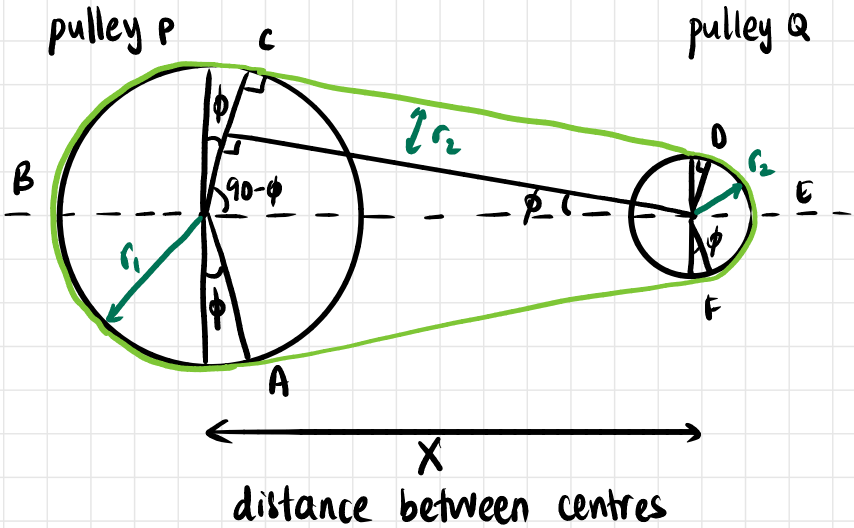
Types of belts

1. Flat
2. V
3. Circular

FLAT BELT DRIVE

Length of Belt

1. Open System



Let r_1 = radius of large pulley
r_2 = radius of small pulley
X = distance between centres

Length of belt $= \overset{\frown}{ABC} + \overline{CD} + \overset{\frown}{DEF} + \overline{FA}$

$$= 2(\overset{\frown}{BC} + \overline{CD} + \overset{\frown}{DE})$$

$$= 2\left[\left(\frac{\pi}{2}+\phi\right)r_1 + X\cos\phi \quad \left(\frac{\pi}{2}-\phi\right)r_2\right]$$

$$= \pi(r_1+r_2) + 2\phi(r_1-r_2) + 2X\cos\phi$$

$$\sin\phi = \frac{r_1 - r_2}{x} \approx \phi$$

$$\cos\phi = (1 - \sin^2\phi)^{1/2}$$

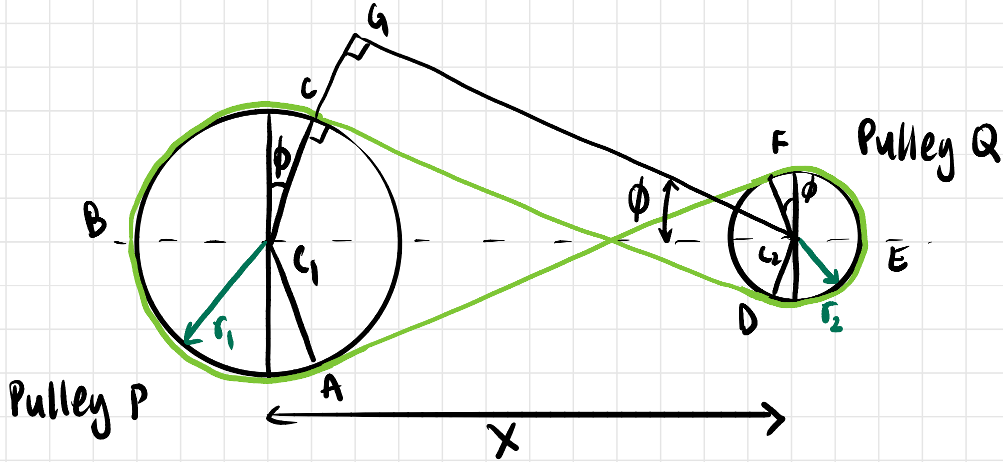
$$\approx 1 - \frac{1}{2}\sin^2\phi$$

$$\approx 1 - \frac{1}{2}\frac{(r_1 - r_2)^2}{x^2}$$

$$\therefore \text{length of belt} = \pi(r_1 + r_2) + \frac{2(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$\boxed{L = \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x}$$

2. Crossed Systems



$$L = \overset{\frown}{ABC} + \overline{CD} + \overset{\frown}{DEF} + \overline{FA}$$

$$= 2\left(\overset{\frown}{BC} + \overline{CD} + \overset{\frown}{DE}\right)$$

$$= 2\left(\left(\frac{\pi}{2}+\phi\right)r_1 + x\cos\phi + \left(\frac{\pi}{2}+\phi\right)r_2\right)$$

$$= \pi(r_1+r_2) + 2\phi(r_1+r_2) + 2x\cos\phi$$

$$\sin\phi = \frac{r_1+r_2}{x} \approx \phi$$

$$\cos\phi = 1 - \frac{1}{2}\sin^2\phi$$

$$= 1 - \frac{(r_1+r_2)^2}{2x^2}$$

$$\therefore L = \pi(r_1+r_2) + \frac{2(r_1+r_2)^2}{x} + 2x - \frac{(r_1+r_2)^2}{x}$$

$$\boxed{L = \pi(r_1+r_2) + \frac{(r_1+r_2)^2}{x} + 2x}$$

Velocity Ratio

$$\frac{\text{driving speed}}{\text{driven speed}}$$

Let d_1 = diameter of driving
d_2 = diameter of driven
N_1 = rpm of d_1
N_2 = rpm of d_2

linear speed of belt = circumferential speed of driving = circumferential speed of driven

$$= \pi d_1 N_1 = \pi d_2 N_2$$

$$\text{velocity ratio} = \frac{N_1}{N_2} = \frac{d_2}{d_1}$$

Effect of thickness of Belt

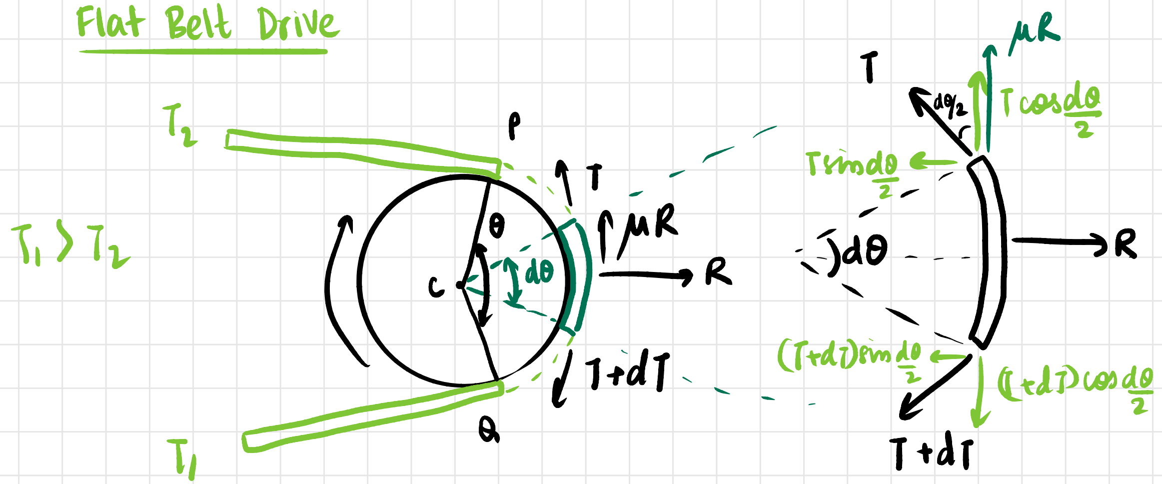
Let t = thickness of belt

$(t/2 + t/2)$

$$VR = \frac{N_1}{N_2} = \frac{d_2 + t}{d_1 + t}$$

Tensions in Belt Drives

Flat Belt Drive



x-direction:

$$R = T\sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2}$$

$$R = T\frac{d\theta}{2} + T\frac{d\theta}{2} + \cancel{dT\frac{d\theta}{2}}^{\,0}$$

$$R = T\,d\theta \longrightarrow (1)$$

y-direction

$$\mu R + T\cos\cancel{\frac{d\theta}{2}}^{\,1} = (T+dT)\cos\cancel{\frac{d\theta}{2}}^{\,1}$$

$$\mu R = dT \longrightarrow (2)$$

(1) & (2)

$$\mu T d\theta = dT$$

$$\mu d\theta = \frac{dT}{T}$$

$$\int_0^{\theta} \mu d\theta = \int_{T_2}^{T_1} \frac{dT}{T}$$

$$\mu\theta = \ln \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = e^{\mu\theta} \qquad (T_1 > T_2)$$

Initial Tension

- Tension when drive not yet in motion
- Once motion starts, increases from T_0 to T_1 on tight end and decreases from T_0 to T_2 on slack end.
- Because the drive does not stretch, the increase in tension at one end = decrease in tension at other end.

$$T_1 - T_0 = T_0 - T_2$$

$$T_0 = \frac{T_1 + T_2}{2}$$

Power Transmitted by Belt Drive

Driving force = diff in tensions

Let v = velocity of drive ($m\ min^{-1}$)

$$\text{Power} = \frac{(T_1 - T_2)v}{60} \quad W$$

$$= \frac{(T_1 - T_2)v}{1000 \times 60} \quad kW$$

Slip in Belt Drives

- In ideal, $dT = \mu R$ or diff. in tension = friction
- Friction sufficient to create motion (prevent sliding)
- If $dT > \mu R$, relative motion between belt and pulley.
- Caused due to low μ (stretch), smooth pulley, ΔT large

Effect of Slip on Velocity Ratio

S_1 = % slip b/w driving pulley and belt
S_2 = % slip b/w driven pulley and belt

$$\text{total } \% S = S_1 + S_2$$

On driving pulley

circumferential speed $d_1 = \pi d_1 N_1$

If belt slips by S_1,

reduced linear speed $= \pi d_1 N_1 \left(\frac{100 - S_1}{100}\right)$

On driving pulley,

circumferential speed $= \pi d_2 N_2$

$$\pi d_2 N_2 = \left[\pi d_1 N_1 \left(\frac{100 - S_1}{100}\right)\right]\left(\frac{100 - S_2}{100}\right)$$

$$= \pi d_1 N_1 \left(\frac{\cancel{100}(100 - S_2 - S_1) + \cancel{S_1 S_2}^{\,0}}{\cancel{100} \times 100}\right)$$

$$= \pi d_1 N_1 \left(\frac{100 - (S_1 + S_2)}{100}\right)$$

$$\pi d_2 N_2 = \pi d_1 N_1 \left(\frac{100 - S}{100}\right)$$

$$\text{velocity ratio} = \frac{N_1}{N_2} = \frac{d_2}{d_1}\left(\frac{100}{100 - S}\right)$$

With thickness of belt,

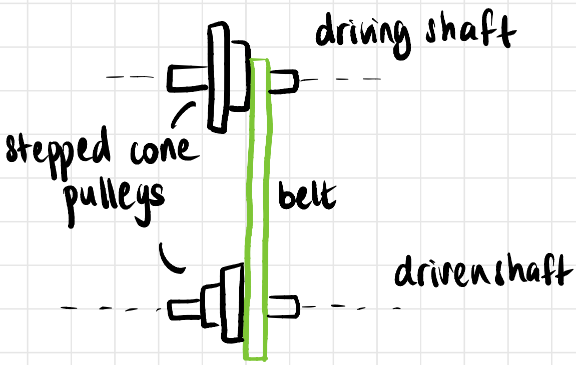
$$\text{velocity ratio} = \frac{N_1}{N_2} = \left(\frac{d_2 + t}{d_1 + t}\right)\left(\frac{100}{100 - S}\right)$$

Creep in Belt Drives

- Due to strech & compression of belt as portions alternate between tight and slack, belt gets stretched
- Results in relative motion, known as creep.

Stepped Cone Pulley

- Frequent changes in speed.
- several pulleys (diff radius) adjacent to each other
- Belt shifts from one pair of pulleys to another to change speeds



Fast and Loose Pulleys

Without starting and stopping main driving shaft, driven shaft can be started and stopped by using fast and loose pulleys

Q: Power is to be transmitted from one shaft to another by means of a belt drive. $d_1 = 600mm$, $d_2 = 300mm$. Distance between centres = 3m.

(a) L = ? for open
(b) L = ? for cross

$r_1 = 300mm$ $= 30 cm$ $r_2 = 150mm$ $= 15 cm$ $x = 300 cm$

(a) Open:

$$L = \pi(r_1 + r_2) + \frac{(r_1 - r_2)^2}{x} + 2x$$

$$= \pi(45) + \frac{(15)^2}{300} + 600$$

$$= 742.12 \text{ cm}$$

(b) Crossed

$$L = \pi(r_1 + r_2) + \frac{(r_1 + r_2)^2}{x} + 2x$$

$$= \pi(45) + \frac{(45)^2}{300} + 600$$

$$= 748.12 \text{ cm}$$

Q: An engine is driving a generator by means of a belt drive. 55cm diameter pulley on the driving shaft runs at a speed of 280 rpm. If radius of pulley on driven shaft is 15cm, determine its rpm.

$d_1 = 55cm$ $\quad$ $r_2 = 15 cm \Rightarrow d_2 = 30 cm$

$N_1 = 280$ rpm $\quad$ $N_2 = ?$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

$$N_2 = 513 \text{ rpm}$$

Q: A motor running at 1750 rpm drives a line shaft at 800 rpm. If the diameter of pulley on driving shaft is 160 mm, determine that of that on the driven shaft.

$N_1 = 1750$ $\quad$ $N_2 = 800$

$d_1 = 160mm$ $\quad$ $d_2 = ?$

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} \Rightarrow d_2 = \frac{N_1}{N_2} \times d_1$$

$$d_2 = 350mm$$

Q: A shaft running at 100 rpm is to drive a coplanar parallel shaft at 150 rpm. Find diameter of the driven pulley if that of the driving pulley is 35 cm. Also determine linear velocity of the belt and velocity ratio.

$N_1 = 100$ rpm $\quad d_1 = 35$ cm
$N_2 = 150$ rpm $\quad d_2 = ?$

$$d_2 = \frac{N_1}{N_2} \times d_1 = 23.33 \text{ cm}$$

linear velocity of belt

$$= \frac{\pi d_1 N_1}{60} = 183.26 \text{ cm s}^{-1}$$

Q: The sum of the diameters of two pulleys connected by means of a belt drive is 900mm. $N_1 = 1400$ rpm, $N_2 = 700$ rpm. Find d_1 & d_2

$$d_1 + d_2 = 900$$

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} \Rightarrow \frac{1400}{700} = \frac{d_2}{900 - d_2}$$

$$2(900 - d_2) = d_2$$
$$1800 = 3d_2 \Rightarrow d_2 = 600$$

$\therefore d_1 = 300$ & $d_2 = 600$

Q: In an open belt drive, the tension on the tight side of the belt is 3000 N. The angle of overlap is measured to be 150°. $\mu = 0.3$. Determine tension on the slack side and the effective pull on the belt.

$T_1 = 3000N \quad \theta = 150° = 5\pi/6 \quad \mu = 0.3$

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$T_2 = T_1 e^{-\mu\theta} = 1367.8 \text{ N}$$

$$\text{effective pull} = T_1 - T_2 = 1632.2N$$

Q: In a crossed belt drive the difference in the tensions between the tight and slack sides is found to be 1000N. If the angle of overlap is 160° and $\mu = 0.3$, determine T_1 and T_2.

$$T_1 - T_2 = 1000$$
$$T_1 = T_2 + 1000$$

$$\theta = 160° = 8\pi/9$$
$$\mu = 0.3$$

$$\frac{T_2 + 1000}{T_2} = e^{0.3 \times 8\pi/9}$$

$$T_2 + 1000 = 2.311\, T_2$$

$$1000 = 1.311\, T_2$$

$$T_2 = 762.67 \text{ N}$$
$$T_1 = 1762.67 \text{ N}$$

Q: In a belt drive with the angle of lap 160° and $\mu = 0.28$. $T_{max} = 50$ N mm^{-1} width, determine initial tension in belt 200mm wide.

$$\theta = 8\pi/9$$
$$\mu = 0.28$$

for 1 mm belt, let $T_1 = T_{max} = 50$ N

$$\frac{50}{T_2} = e^{0.28 \times 8\pi/9}$$

$$T_2 = 50 \times e^{-0.28 \times 8\pi/9} = 22.88 \text{ N}$$

for 200 mm wide belt,

$$T_1' = 50 \times 200 = 10{,}000 \text{ N}$$

$$T_2' = 22.88 \times 200 = 4575 \text{ N}$$

$$T_0 = 7287.66 \text{ N}$$

Q: In a belt drive system, the driven pulley with 400mm diameter runs at 200 rpm. If angle of lap is 165°, $\mu = 0.25$, determine power transmitted by the belt drive if initial tension should not exceed 10 kN

$d_2 = 400$mm $\quad \theta = 11\pi/12 \quad T_0 = 10$ kN

$N_2 = 200$ rpm $\quad \mu = 0.25$

$T_1 = 10 + x$
$T_2 = 10 - x$

$$\frac{10 + x}{10 - x} = e^{\mu\theta} = 2.054$$

$$10 + x = 20.54 - 2.054x$$

$$3.054x = 10.54$$

$$x = 0.2897 \text{ kN}$$

$T_1 = 10 + x$, $T_2 = 10 - x$ $\Rightarrow$ $T_1 - T_2 = 2x$

$$T_1 - T_2 = 2 \times 0.2897 \text{ kN}$$

v = velocity of belt = $\dfrac{\pi d N}{60} = 4.189 \text{ ms}^{-1}$

power = $(T_1 - T_2)v$

$= 2 \times 0.2897 \times 4.189$
$= 2.427$ kW

Advantages & Disadvantages of Flat Belt Drives

Advantages

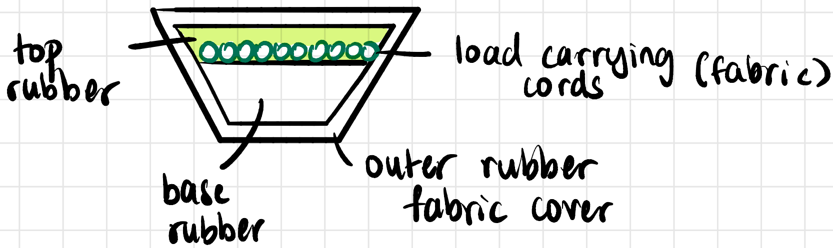
1. Small bending cross-section ⇒ small bending loss or higher efficiency (upto ~98%)
2. Simple & secure installations
3. More contact area ⇒ less slip (< 1%)
4. More durable than V belts
5. Suitable for large centre distances
6. Economical

Disadvantages

1. Not suitable for small centre distances (low θ and low power transmission)
2. Velocity ratio cannot be maintained exactly
3. Loss due to slip and creep
4. Cannot transmit high power effectively.

V-BELT DRIVES

- Trapezoidal cross-section, run in V grooves of pulley
- Rubber reinforced with fibrous material
- Wedging action — higher power



Advantages and disadvantages

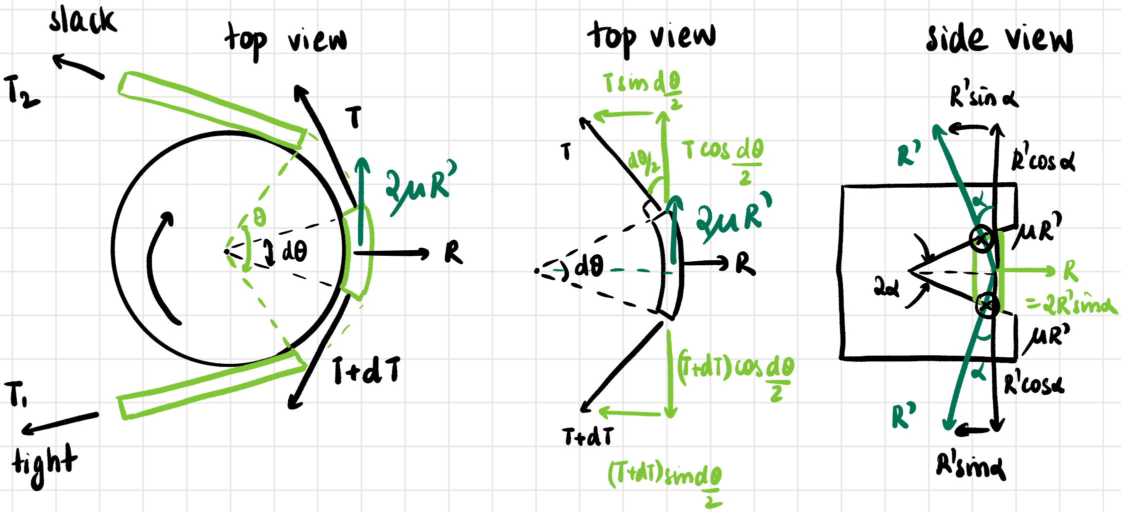
Advantages

1. High power transmission
2. Small centre distances
3. High VRs
4. No slip
5. Even if one of the belts snap transmission may continue temporarily
6. Shaft axes in any position
7. Several machines from single driving shaft.

Disadvantages

1. Complex construction
2. Less durable
3. Not for large centre distances
4. Expensive

Ratio of Tensions



Vertical direction

$$2\mu R' = dT \cos\cancel{\frac{d\theta}{2}}^{\,1}$$

$$R' = \frac{dT}{2\mu} \longrightarrow (1)$$

Horizontal

$$T\sin\frac{d\theta}{2} + (T+dT)\sin\frac{d\theta}{2} = 2R'\sin\alpha$$

$$T\,d\theta = 2R'\sin\alpha$$

$$R' = \frac{T}{2} \operatorname{cosec}\alpha \, d\theta \quad -(2)$$

From (1) and (2)

$$\frac{dT}{\cancel{2}\mu} = \frac{T}{2} \operatorname{cosec}\alpha \, d\theta$$

$$\frac{dT}{T} = \mu \operatorname{cosec}\alpha \, d\theta$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \mu \operatorname{cosec}\alpha \int_0^{\theta} d\theta$$

$$\ln\left(\frac{T_1}{T_2}\right) = \mu \operatorname{cosec}\alpha \, \theta$$

$$\frac{T_1}{T_2} = e^{\mu\theta \operatorname{cosec}\alpha}$$

Q: A V-belt drive, $P = 8000\,W$, $N = 300$ rpm, $\alpha = 20^\circ$, $\theta = 160^\circ$, $d = 500mm$, $\mu = 0.5$, $T_1, T_2 = ?$

$$P = \frac{(T_1 - T_2)\pi d N}{60}$$

Let $T_1 = T_0 + x$
$T_2 = T_0 - x$

$\therefore T_1 - T_2 = 2x$

$$P = \frac{2x \pi d N}{60}$$

$$8000 = \frac{2x \pi (0.5)(300)}{60}$$

$x = 509.30$ N

$$\frac{T_0 + 509.30}{T_0 - 509.3} = e^{(0.5)(\operatorname{cosec} 20)(8\pi/9)}$$

$$= e^{(\operatorname{cosec} \alpha)(4\pi/9)}$$

$$= 59.2877$$

$T_0 + 509.30 = 59.2877 T_0 - 30195.202$

$30704.502 = 58.2877 T_0$

$T_0 = 526.78$

$T_1 = 1036.08$ N
$T_2 = 17.48$ N

Gear Drives

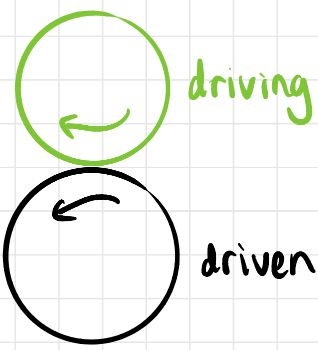
- Exact velocity ratio; no slip (positive drives)
- Short centre distance
- Lubrication necessary
- Noise and vibrations
- Production cost high

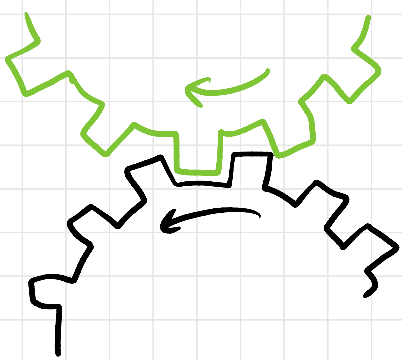
Type of Gear Drives

1. Spur gears
2. Helical gears
3. Bevel gears
4. Elliptical gears
5. Worm and worm wheel gear
6. Rack and pinion gear

Spur Gears

- parallel & coplanar axes of shafts
- teeth of gear wheels parallel to axes
- higher power
- noise high
- machine tools and automobiles

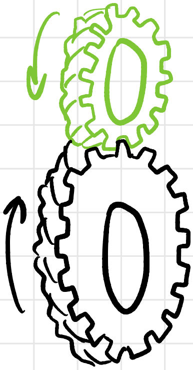


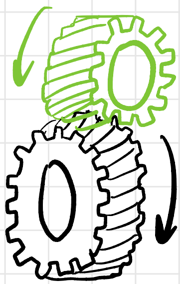


Helical Gears

- similar to spur but teeth cut in the form of helix
- parallel, non-parallel, non-intersecting shaft
- progressive tooth contact
- low noise
- disadvantage – end thrusts
- automobile power

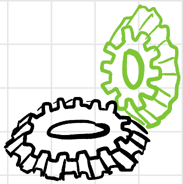
also called spiral





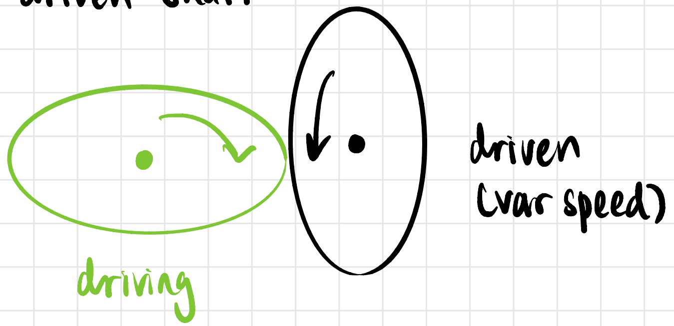
Bevel Gears

- intersecting axes
- teeth cut on conical surfaces
- equal sizes & perpendicular axes – Miter Gears



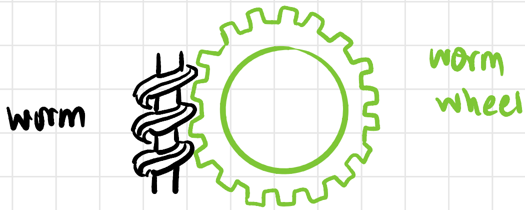
Elliptical Gears

- 2 equal sized elliptical gears meshed
- to obtain varying rate of speed in each revolution of driven shaft



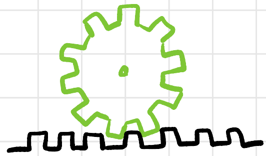
Worm and Worm wheel

- right angles and non-coplanar axes
- worm (screw) with threads and worm wheel
- helical threads
- ~60:1 VR (large speed reduction)

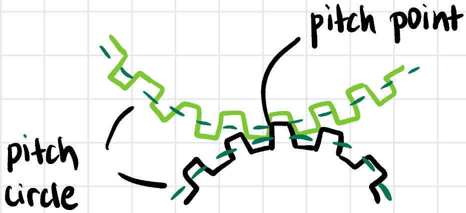


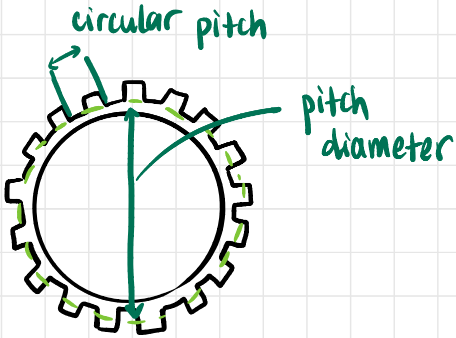
Rack and Pinion

- rotary to linear motion



Gear Nomenclature





d= pitch diameter
T = no. of teeth on gear wheel

$$\text{module} = \frac{d}{T}$$

diametral pitch $P_D = \frac{T}{d}$

circular pitch $P_c = \frac{\pi d}{T}$

Velocity Ratio of Gear Drives

linear speed of pitch cylinder representing driving gear = linear speed of pitch cylinder representing driven gear

$$\pi d_1 N_1 = \pi d_2 N_2$$

circular pitch of gears must be the same

$$p_c = \frac{\pi d_1}{T_1} = \frac{\pi d_2}{T_2}$$

$$\frac{N_1}{N_2} = \frac{d_2}{d_1} = \frac{T_2}{T_1}$$

Velocity ratio for worm and worm wheel

$$VR = \frac{\text{RPM of the worm}}{\text{RPM of worm wheel}} = \frac{\text{no. of teeth on worm wheel}}{\text{no. of threads on worm}}$$

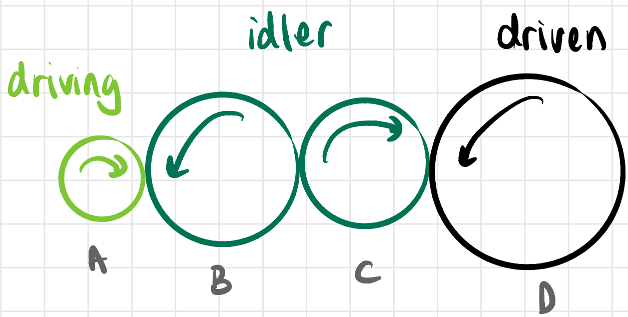
Gear Train

- Arrangement of number of successively meshing gear wheels for power transmission

Simple Gear Train

- series of wheels mounted on shafts
- one gear per shaft
- intermediate (idler) gears

- even no. of idler gears: opposite direction
- odd no. of idler gears: same direction

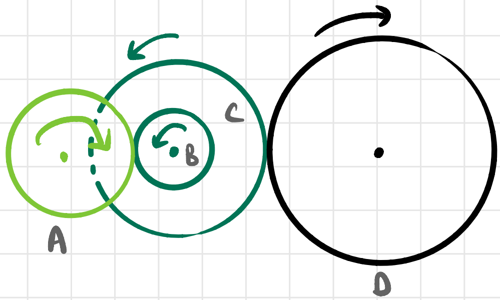


$$VR = \frac{N_A}{N_D} = \frac{T_D}{T_A}$$

$$\text{train value} = \frac{1}{VR} = \frac{T_A}{T_D} = \frac{N_D}{N_A}$$

Compound Gear Train

- Multiple gears on single shaft



velocity ratio

A drives B

$$\frac{N_A}{N_B} = \frac{T_B}{T_A}$$

B and C same speed

$$N_C = N_B = N_A \frac{T_A}{T_B}$$

C drives D

$$\frac{N_C}{N_D} = \frac{T_D}{T_C} \Rightarrow \frac{N_A}{N_D}\frac{T_A}{T_B} = \frac{T_D}{T_C}$$

$$VR = \frac{N_A}{N_D} = \frac{T_B}{T_A} \times \frac{T_D}{T_C}$$

$$\text{train value} = \frac{T_A}{T_B} \times \frac{T_C}{T_D}$$

Mechatronics

- mechanics + electronics
- mechanical electronic systems; synergistic combination of mechanical, electrical, electronics and computer engineering

refer ppt